

HW 9

Due Friday, 11/6

1. Solve Problem 15.22.
2. Solve Problem 15.23.
3. Let X be a set, and let $A \subset X$ be a subset. Consider the function $D: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ defined by $D(B) = A \triangle B$. (Recall that the symmetric difference $A \triangle B$ is defined by $A \triangle B = (A \setminus B) \cup (B \setminus A)$.)
 - (a) Is the function D one-to-one for all sets $A \subset X$? If so, prove it; if not, give an example of sets $A \subset X$ for which D is not one-to-one.
 - (b) Is the function D onto for all sets $A \subset X$? If so, prove it; if not, give an example of sets $A \subset X$ for which D is not one-to-one.
 - (c) Is the function D a bijection for all sets $A \subset X$? If so, prove it; if not, give an example of sets $A \subset X$ for which D is not one-to-one.
4. Prove Theorem 16.7 from the text.
5. Solve Problem 16.14.
6. Let $n > 1$ be an integer. For each $x \in \mathbb{Z}$, we can define a function $a_x: \mathbb{Z}_n \rightarrow \mathbb{Z}_n$ by $f([k]_n) = [x + k]_n$. Prove that a_x is always a bijection. (Here $\mathbb{Z}_n$ is the set of equivalence classes of integers modulo n .)

Additional Suggested Problems: 15.13, 15.24, 15.26, 16.4, 16.5, 16.21

Extra Credit: Let $f: X \rightarrow X$ be a function such that f is onto.

- a) Prove that if $f \circ f \circ f = f$, then f is a bijection (this is Problem 16.22).
- b) Prove that if $f \circ f = f$, then f is the identity function i_X .
- c) Give an example of a function $f: X \rightarrow X$ such that f is onto and satisfies $f \circ f \circ f = f$, but $f \neq i_X$.